# POZNAN UNIVERSITY OF TECHNOLOGY



#### EUROPEAN CREDIT TRANSFER AND ACCUMULATION SYSTEM (ECTS)

## **COURSE DESCRIPTION CARD - SYLLABUS**

Course name Ordinary differential equations [S1MwT1>RRZ]

Course			
Field of study Mathematics in Technology		Year/Semester 2/3	
Area of study (specialization) –		Profile of study general academi	с
Level of study first-cycle		Course offered ir polish	1
Form of study full-time		Requirements compulsory	
Number of hours			
Lecture 30	Laboratory class 0	es	Other (e.g. online) 0
Tutorials 30	Projects/seminar 0	S	
Number of credit points 4,00			
Coordinators		Lecturers	
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#### **Prerequisites**

Basic knowledge of linear algebra and mathematical analysis.

#### **Course objective**

To introduce the basic concepts and theorems of the theory of differential equations, to familiarise students with the basic methods used in finding and analysing the properties of solutions of ordinary differential equations.

#### Course-related learning outcomes

Knowledge:

Student:

1. knows most of the basic definitions and theorems of the general theory of differential equations [K\_W01 (P6S\_WG)],

2. knows the connections of issues in the theory of differential equations with other branches of theoretical and applied mathematics [K\_W03 (P6S\_WG)], [K\_W09 (P6S\_WG)],

3. knows and understands to an advanced degree the theory of mathematics necessary to understand the theory of numerical methods [K\_W15 (P6S\_WG)].

Skills:

Student:

1. is able to solve basic types of ordinary differential equations [K\_U01 (P6S\_UW)],

2. is able to solve a system of linear ordinary differential equations [K\_U01 (P6S\_UW)],

3. be able to reduce a higher order differential equation to a system of differential equations of order one [K\_U01 (P6S\_UW)],

4. knows how to find the fundamental matrix of a system of linear equations [K\_U01 (P6S\_UW)], 5. knows what a stationary point is and knows the definition of asymptotic stability of a stationary point and stability in the sense of Lyapunov and knows how to investigate the stability of a stationary point [K\_U01 (P6S\_UW)],

6. is able to build and analyse simple mathematical models [K\_U03 (P6S\_UW)].

Social competences:

Student:

1. understands the importance of ordinary differential equations as a tool for formulating the laws of nature [K\_K06 (P6S\_KK)],

2. knows the limitations of his/her own knowledge and understands the need for further education [K\_K01 (P6S\_KK)],

4. is able to independently search for information in literature, also in foreign languages [K\_K05 (P6S\_KK)],

5. is ready to fulfil his/her social role as a graduate of a technical university, including communicating popular science content to the society, as well as identifying and solving basic problems concerning the field of study and promoting mathematics as a basis for analytical reasoning and precise formulation of correct conclusions [K\_K12 (P6S\_KR)].

### Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Learning outcomes presented above are verified as follows:

Lecture - written exam (pass mark 50%)

Exercises - two written colloquia (pass mark 50%)

### Programme content

1. Introductory concepts: definition of a differential equation of order n, its solution, integral curve; general solution, special solution and singular solution. Examples of problems leading to differential equations.

2. Separated variable equation and equations reducible to it: definition of a separated variable equation, Cauchy"s problem and its geometric interpretation, existence and uniqueness theorems for solutions of a separated variable equation, homogeneous equation: definition, existence and uniqueness theorems for solutions, solution method; solving equations of the form y' = f (ax + by + c).

3. First-order linear differential equation: definition of linear equation, theorem of existence and uniqueness of solution, methods of solving non-homogeneous equations: method of variation of parameters, method of undetermined coefficients.

4. Nonlinear differential equations reducible to linear equations: Bernoulli equation, Riccati equation; definitions, solution methods.

 Lagrange and Clairaut equations, definitions, solution methods; geometric interpretation of the solution of the singular Clairaut equation, example of application of the Clairaut equation in geometry.
Complete differential equation: definition of differential equation, existence theorem of solution, general integral of complete equation, incomplete differential equation, integral factor depending on one variable.

7 The existence and uniqueness theorems for the solution of the Cauchy problem of a system of differential equations (Picard's theorem, Peano's theorem). Picard's method of approximate solution of differential equations.

8. Differential equations of second order: definition of a differential equation of second order, Cauchy initial problem for an ordinary differential equation of second order, boundary condition and their geometric interpretations; methods of solving differential equations of second order reducible to equations of first order (differential equation not containing the sought function, equation not containing the independent variable, homogeneous equation with respect to the sought function and its

derivatives).

9. Linear differential equations of higher orders: existence and uniqueness theorem for the solution of an initial problem, Wronsky matrix, Wronskian, fundamental system of solutions, necessary and sufficient condition for linear independence of a system of solutions; general solution of a linear homogeneous equation, linear homogeneous equation with constant coefficients, characteristic equation; prediction principle for linear nonhomogeneous equations of higher orders with constant coefficients; method of variation of parameters for nth order nonhomogeneous linear differential equations; Euler differential equations.

10. Laplace transform: definition of Laplace transform and its basic properties; derivation of formulas for Laplace transforms of more important functions; inverse Laplace transform; application of Laplace transform to the solution of linear differential equations.

11. Systems of first-order linear differential equations: the existence and uniqueness theorem for the solution of the Cauchy problem; solution of systems of homogeneous linear equations with constant coefficients (Euler's method, elimination method).

### **Teaching methods**

Lecture with multimedia presentation supplemented with examples given on the blackboard, theory presented in connection with the students" current knowledge.

Exercises - two colloquia evaluating practical skills of solving tasks and current evaluation of student"s work during classes.

### Bibliography

Basic

1. M. Gewert, Z. Skoczylas, Równania różniczkowe zwyczajne. Teoria, przykłady, zadania,Oficyna Wydawnicza GiS, Wrocław 2016.

2. W. Krysicki, L. Włodarski, Analiza matematyczna w zadaniach, t. 2, Wydawnictwo Naukowe PWN, Warszawa 2022 (wydanie 27).

3. A. Palczewski, Równania różniczkowe zwyczajne (teoria i metody numeryczne z wykorzystaniem komputerowego systemu obliczeń symbolicznych), Wyd. Naukowo-Techniczne, Warszawa 2004. Additional

1. J. Mikołajski, Z. Sołtysiak, Zbiór zadań z matematyki dla studentów wyższych szkół technicznych, cz.III, Kalisz 2010.

2. I. Foltyńska, Z. Ratajczak, Z. Szafrański, Matematyka dla studentów uczelni technicznych, Wydawnictwo PP 2004.

### Breakdown of average student's workload

	Hours	ECTS
Total workload	100	4,00
Classes requiring direct contact with the teacher	63	2,50
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	37	1,50